DESIGN OF EXPERIMENTS and ROBUST DESIGN

Problem solutions in design and productions environemts often require experiments to find a solution. Design of experiments are collection of statisical methods properly used maximize the probability of finding the best solution at the lowest cost.

The fundamental idea is to vary a problem related factor and study changes in process or product performance. To increase the experment information content, two or more factors are varied at the same time. The concerted factor variations are specified by a matrix with carefully selected characteristics. Two or more matrixes are ofthen used in series in iterative steps to find the best solution.

Design of experiments is used to identify important factors, to optimize physical and virtual systems and to create robust products and processes.

Many industrial businesses do not use the powerful resource of statistically designed experiments – in most cases due to unsufficient knowledge of the methods potential.

Long experience from Japan and recent experience from six sigma programs shows that systematic utilization of designed experiments often produces large dividends on a relatively modest investment.

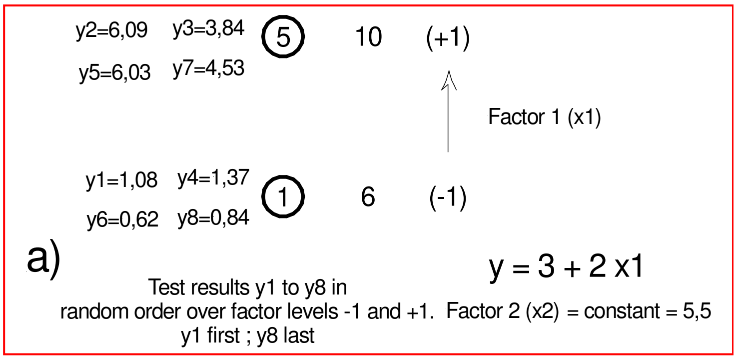
Theoretical foundation

The theorethical foundation of designed experiments is mathematical statistics. The theoretical part of designed experiments include basic models such as density- and probabilityfunctions, statistical distributions, sample theory; intermediate and more advanced models such as regression analysis and ANOVA, matrix based statstics; and application specific statistical tools. Simpler applications of design experiments do not require an extensive knowledge in theoretical statistics. It requires a good knowledge of basic statistical relationships and what assumptions these are based on.

Factorials

Design of experiments are best illustrated by example. Let us assume we have problem with the performance of a product / process we are developing – for example high emissions from a boiler in a powerplant, low structural yield in a laser weld or to large variations in film thinckness in a semiconductor production plant.

Let us further assume that we have found two factors that probably can solve the problem.
We will show two optional routes to create an idea of how factors affect performance.
We start with a traditional experimental design investigating "one factor at a time". Figur a) shows an experiment where a first factor (x1) is investigated at two levels. We start with adjusting x1 to 6 units and then read the performance value y1 = 1,08 then repeat the testprocedure adjusting x1 to 10 units and read y2 = 6,09. The second factor x2 has been set at a constant value during the tests. The measurments have been repeated in y3 to y8 to create sufficient precision. The levels are usually coded to create efficient statistical models. If we in this case use (x1- 8)/2 the transformation of the low level 6 is -1 and the high level 10 is +1.



The next step is to estimate a function between the factor settings and the product / process performance. This is usually done with appropriate regression software. A regression analysis of the example data gives the model y = 3 + 2 * x1 where y is measured performance and x1 is factor adjustment in coded scale -1 to +1. If the model is calculated with the coded values -1 and +1 inserted the result is y = 1 and y = 5 respectively, the values in the lower and upper rings in figure a). These values coinside with the mean values in this case.

We notice that this result did cost us 8 tests and that an estimate of the second factor x2 vill cost us 8 more tests.

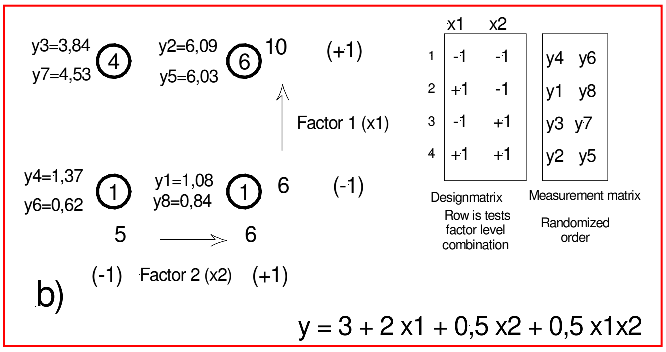
If we assume that our project budget is limited to 8 tests and we want to estimate both factors we can distribute the test in a factorial as illustrated in figure b). The figure shows four rings arranged in a square with vertical setting directon for x1 and horisontal setting direction for x2.

Both factors have been coded as low level -1 and high level +1. Test points are marked with circles and correspond to the code combinations lower left point {-1,-1}, lower right point {-1, +1} etc. The 8 test points have been distributed evenly over the four points i.e. two points per point. The test are performed in random order as y index indicate – 1is the first, 2 the second etc in run order. The design can alternatively be written on a matrix form as illustrated in the left side of figure b). Row 1 corresponds to the lower left point, row 2 lower right point etc. Figure b) shows two matrixes – the left is the factorial design and the right is the measurement matrix.

A regression analysis of the data result in

y = 3 + 2 x1 + 0,5 x2 + 0,5 x1x2.

where x1 and x2 are in coded units. The model is telling us that one unit increase in x1 on average double the performance y. And one unit increase in x2 on average increase the performance y with 50 %. The term 0,5 x1x2 is called interaction and is a measurment of how much the setting of x1 influence the effect of x2 or vice verse. In this case x2 has no influence on performance if x1 is set at -1 but has a positive effect if x1 is set at +1. A study of the mean values inscribed in the four circles in figure b) verify this.



A comparison of the same budget experiments reveal that a) deliver one effect while b) deliver three effects. In addition the a) result is only valid at the fixed value of the other factor x2 while the b) result in most cases is valid for all {x1, x2}coordinates of the design area and its vicinity.

Case a) has compared to case b) a slightly better precision in estimates. This due to more degrees of freedom for error estimates. However, compared to other information generated in a “real world” experiment small precision diffrences are usually of little practical consequence.

We have briefly shown some advantages of chosing factorials instead of traditional “one factor at a time” designs. Generally factorials gives *better economy*, *estimates of interactions* and more *generally valid results* with *better repeatability*. When used in an optimizing scenario *they reduce the risk of suboptimizing*.

The number of factors can theoretically be any number. However, the number of tests in a two level full factorial design is determined by 2^k wher *k* is the number of factors. This will quickly lead to a very large number of tests not manageble whitin a normal project budget. Fractional factorals have been developed to limit experiment sizes when investigating a large number of factors.

Factors (x) and designs

Designed experiment factors x may have very different charachteristics. In our two factor example the factors where continuous unrestricted and varied at two levels. Depending on scenario the factors may be proportions / mixtures (sum of factor levels is 1), have discrete levels (e.g. different catalysts), have more than two levels etc. A matrix may include many different types of factors. Factor types and the goal of the experiment shape the final design matrix specification. Usually matrices are of standardtype – as in figur b) – or generated through computer algorithms.

System response (y)

Also system responses such as product or product performance can have many different characteristics. They may be continuous or catergorical, be defined as positive numbers, be proportions or frequencies. In robust design you usually work with location and dispersion responses. In reliablitity experiments you usually work with time or number of cycles to failure where tests often are intentionally interrupted before all objects have failed (censuring). It is very common to use more than one response, often of different types, in the same experiment.

Evaluation and estimation methods

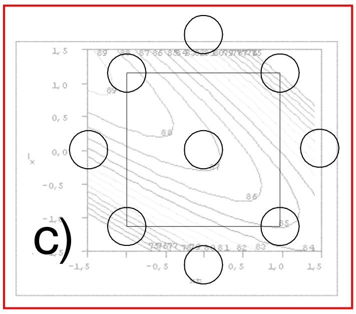
Analysis of experiments based on factorials is a *knowledge building process* where data such as technical information from product /process, computer plots and estimates are synthesized. Estimation method is chosen based on factor and response characteristics. Often a least square (regression) or in more advanced cases, a Maximum Likelihood method is chosen.

Fractional factorials

It can be shown that factorials generate necessary information already at a small carefully selected set of the tests in a full factorial. Designs created in this way are called fractional factorials. Fractional factorials with close to one test per factor with 20 and more factors do occur. Fractional factorials are often used as a screeningtool when active factors and interactions are unknown. The results from fractionated experiments are often ambiguous and require further tests or validation for an unambiguous result. Most factorials performed in industry are fractional factorials.

Response surface analysis

With designs of the type shown in figure b) is used to estimate the second order model y = b0 + b1x1+ b2x2 + b12x1x2. Ofthen you want an estimate of the complete second order model . y = b0 + b1x1+ b11 $x1^2$ + b2x2 + $b22x2^2$ +b12x1x2. This requires a complementary addition to the design in figure b) or a copletly new design. Figure c) shows a design where b) has been complemented with a center point and four start points.


c)

Experiments based on this design may be used to find extreme values of different types. Sometimes the investigated area is far from the extreme point of interest. Special designs for the most efficient move can then be used. When the new area of interest has been found a new response surface design (figure c)) can be used to get a more detailed surface estimate.
Response surfaces with more than two factors are difficult to visualize. It is therefore common practice to use eigenvalue analysis for surface identification and dimension reduction. Eigenvalue analysis can also be used to search for mechanisms and probable mechanistic models.

Robust design

As designers we are not capable of controling all the paremeters and variation sources influencing our products functions. A number of factors are active – from variation in components from subcontractors, via variation in production process to the product working environment and variation in customer usage. The goal of robust design is to find the combination of product parameter setting that is immune to these noise spectra. Experiments are often the most efficient way of finding these settings.

Figure d) illustrates a robust design experimental design. We strat with dividing the factors in control factors chosen among the factors the designer can control and noise factors chosen among factors thar are not control factors and with the highest probability to interfere with the product functions.

	x1	x2	x3: -1	x3: +1	
1	-1	-1	y2	y7	ym1; s1
2	+1	-1	y4	y8	ym2; s2
3	-1	+1	y5	y3	ym3; s3
4	+1	+1	y6	y1	ym4; s4

{x1,x2} = control factor
{x3} = noise factor

d)

The design matrix is then divided in two parts – the inner control matrix and the outer noise matrix. The experiment is then performed run by run until the measurement matrix is filled with data. Mean *ym* and variance s^2 for each control factor level combination is the estimated followed by an estimate of control factor models y = f(x) of mean and variance and finally a search for best control factor setting. An alternative metod without the modelling stage is sometimes used.
Both static functions e.g. a ball bearing fixation of a wheel as dynamic functions e.g. the break moment of a break system as a function of different pedal pressures can be given a robust design with this method.

Design of experiments and quality

Statistical methods and desing of experiments has always been a vital part of japanese quality systems. This is also true for the lately in industry more commonly applied six sigma and design for six sigma programs (DFSS).

Applications

Design of experiments are a well established tool within the big companies six sigma efforts and are commonly used in other contexts. Projects with profits of more than 1 million euro in projects with desing of experimetns as key component are not uncommon. XR TECH has performed undustrial projects with a 5 year profit of 1 million euro and 3 month return on investment. In other cases the experiments yield other type of benefits e.g. projects improving product specifications leeding to a substantial lower tender risk on the supplier side.

XR TECH services

XR TECH has supplied courses, software and project support services since 1987. For more information please contact Jan Åkerström at XR TECH - 0709 – 210 686 or at jan.akerstrom@xrtech.com.